Topological stability of the half-vortices in spinor exciton-polariton condensates

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Half-vortices have been recently shown to be the elementary topological defects supported by a spinor cavity exciton-polariton condensates with spin-anisotropic interactions [Y. G. Rubo, Phys. Rev. Lett. **99**, 106401 (2007)]. A half-vortex is composed by an integer vortex for one circular component of the condensate, whereas the other component remains static. We analyze theoretically the effect of the splitting between TE and TM polarized eigen modes on the structure of the vortices in this system. For TE and TM modes, the polarization states depend on the direction of propagation of particles and impose some well-defined phase relation between the two circular components. As a result elementary topogical defects in this system are no more half-vortices but integer vortices for both circular components of the condensate. The intrinsic lifetime of half-vortices is given and the texture of a few vortex states is analyzed.

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I. INTRODUCTION

Interactions between quantum particles lie behind a number of intriguing phenomena in the field of condensed-matter physics. Being treated within mean-field approximation, for a system of interacting bosons they result in a nonlinear term in the Gross-Pitaevskii equation, which is currently routinely used for the description of dynamics of Bose-Einstein Condensates (BECs) of cold atoms.[1] A similar equation, known as nonlinear Schrödinger equation, is widely used in nonlinear optics for description of such phenomena as self-focusing of laser beams and propagation of solitons.[2]

The fields of BEC and nonlinear optics meet each other in the context of planar semiconductor microcavities: the mesoscopic objects designed to enhance the light-matter interaction. A microcavity consists of a pair of distributed Bragg mirrors confining an electromagnetic mode and one or several quantum wells (QWs) with an excitonic resonance, which are placed at the antinodes of the electric field. In strong coupling regime, where coherent exciton-photon interaction overcomes the damping caused by the finite lifetime of excitons and cavity photons, a new type of elementary excitations, called exciton polaritons (or cavity polariton), appears in the system. The polaritons are a mixture of material excitations (excitons) with light (photons).

The hybrid nature of polaritons gives them a set of peculiar properties. First, at relatively small densities, polaritons exhibit bosonic properties.[3] Second, due to the presence of a photonic component, the effective mass of the polaritons is extremely small (10^{-4}–10^{-5} of the free electron mass) while the presence of an excitonic component makes possible efficient polariton-polariton and polariton-phonon interactions. These properties make possible polariton Bose condensation[4] suggested more than 10 years ago, up to high temperatures.[5,6] The simulations have shown that relaxation of polaritons can become faster than their radiative lifetime, allowing the formation of a quasiequilibrium polariton gas. These predictions have been confirmed by the recent observation of polariton condensation[7–14] and the demonstration of the thermodynamic regime[13,14] where the behavior of the polariton gas is well described by its thermodynamic variables (temperature and chemical potential). The next step after the observation of the condensation itself is to study the dynamical properties and the specificities of polariton condensates. One of the important properties is superfluidity. The phase transition expected for two-dimensional polaritons is rather a Berezinskii-Kosterlitz-Thouless (BKT) transition toward a superfluid state[15] and not the BEC. Such a phase transition has not been immediately observed in CdTe-based and GaN-based structures, because of the presence of a structural disorder, which has led to the formation of an Anderson Glass phase[16] or to the condensation in a single in-plane potential trap.[17] Only in a cleaner GaAs-based sample some signatures of BKT phase transition have been reported.[18] If this observation is confirmed, it would rule out the claims that no superfluid behavior can be achieved in a system of particles showing a finite lifetime.[19,20] Another way to excite a superfluid flow of polaritons is to properly design a resonant excitation experiment, as described theoretically[21,22] and recently evidenced experimentally.[23] In this framework the study of fundamental properties of polariton vortices is of a strong interest. On one side, the BKT transition between normal and superfluid states in two-dimensional system is closely connected with the formation of topological defects (vortex-antivortex pairs). On the other side, the recent growth of the experimental activity devoted to cavity polariton condensates opened a race to the observation of exotic phenomena. Observation of a vortex pinned to a defect in a disordered cavity has been reported,[24] whereas the formation of a lattice of vortices in a potential trap has been predicted theoretically in the scope of a Ginzburg-Landau model.[25] In these two works the peculiar spin structure of polaritons was not taken into account. In fact, only one theoretical work did study vortex states in homogeneous spinor polariton condensates.[26] In this work Rubo shows that elementary polariton vortex states are the so-called "half-vortices." They are characterized by a half-quantum change in the phase of the condensate, i.e., the phase of the wave function is changed by $\pm\pi$ after encircling the point of singularity. This analysis however was not considering the facts that polariton eigen states in a microcavity are normally TE or TM polarized,[27] with a finite-energy splitting between these two states (TE-TM splitting).

In the present work we therefore consider the impact of the TE-TM splitting on polariton vortices. We first show that using the basis of circularly polarized states (contrary to the basis of linearly polarized states used by Rubo[26]) allows to describe a half-vortex as one vortex for one circular component, whereas the other circular component remains immobile. We then show that the TE-TM splitting couples the half-vortices of opposite circularity, which cease to be the stationary solutions of the spinor Gross-Pitaevskii equations. The elementary (stationary) excitation of the condensate with the TE-TM splitting is composed by one vortex of each circular component. This result does not mean that the half-vortices cannot be observed experimentally, but rather that they should decay in time and therefore should not be used for the calculation of the critical temperature of the BKT phase transition. The paper is organized as follows. In the second section we discuss in detail the spin structure of cavity polaritons. In the third section the polarization structure of polariton vortices is analyzed. Results and discussions are presented in the fourth section. The fifth section draws the main conclusions.

II. SPIN STRUCTURE OF CAVITY POLARITONS

An important peculiarity of cavity polaritons is linked with their spin structure. Like other bosons, polaritons exhibit an integer spin, inherited from spins of excitons and photons. In QWs the lowest energy level of a heavy hole (having a spin $S_z=3/2$) lies typically lower than any light-hole level ($S_z=1/2$) and thus the entire exciton spin in a QW has projections $S_z=\pm 2, \pm 1$ on the structure growth axis. The states with $S_z=\pm 2$ are not coupled to light and thus do not participate in polariton formation. As they are split off in energy, normally they can be neglected while considering polariton dynamics.[28] On the contrary, states with $S_z=\pm 1$ form the optically active polariton doublet and can be created by σ_+ and σ_- circularly polarized light, respectively. Thus, from the formal point of view, the spin structure of cavity polaritons is analogical to spin structure of the electrons (both being two-level systems), which permits to introduce the concept of a pseudospin vector $\vec{S}$ for the description of their polarization dynamics.[29] The latter is determined as the coefficient of the decomposition of the 2×2 spin-density matrix ρ of polaritons on a set consisting of the unity matrix $\mathbf{I}$ and three Pauli matrices $\sigma_{x,y,z}$,

$$\rho = \frac{N}{2}\mathbf{I} + \vec{S}\cdot\vec{\sigma}, \tag{1}$$

where N is the total number of particles. The orientation of the pseudospin completely determines the polarization of the emission from a microcavity. According to a generally accepted convention, orientation of the pseudospin along z axis corresponds to circular polarized emission while pseudospin lying in x-y plane corresponds to linear polarized emission.

The spin dynamics of cavity polaritons has become a field of intense research since 2002.[30] It is governed by two factors. First, at $\vec{k}\neq 0$ there is an effective in-plane magnetic field which results in the pseudospin rotation manifesting itself in the oscillations of the polarization degree of photoemission in the time domain. It is well known that due to the long-range exchange interaction between the electron and hole, for excitons having nonzero in-plane wave vectors, the states with dipole moment oriented along and perpendicular to the wave vector are slightly different in energy.[31] In microcavities, the TE-TM splitting of polariton states is greatly amplified due to the exciton coupling with the cavity mode, which is also split in TE and TM polarizations.[27] An important feature of the effective magnetic field generated by the TE-TM splitting is the dependence of its direction on the direction of the wave vector: it is oriented in the plane of the microcavity and makes a double angle with the x axis in the reciprocal space,

$$\vec{\Omega}_{eff}(k) \sim \mathbf{e}_x \cos(2\phi) + \mathbf{e}_y \sin(2\phi). \tag{2}$$

This peculiar link between the orientation of the effective magnetic field and polariton wave vector leads to remarkable effects in the real-space dynamics of the polarization in quantum microcavities, including the optical spin Hall effect,[32] possible formation of polarization patterns,[33] and creation of polarization vortices.[34]

Second, polariton-polariton interactions are known to be spin anisotropic. Since the exchange interaction plays a major role, the interaction of polaritons with parallel spin projections on the structure growth axis is much stronger than that of polaritons with antiparallel spin projections.[35] This leads to a mixing of linearly polarized polariton states, manifesting itself in remarkable nonlinear effects in polariton spin relaxation, such as self-induced Larmor precession and inversion of linear polarization upon scattering.[36]

In the domain of polariton BEC, spin properties of cavity polaritons play a major role. It was argued that under unpolarized nonresonant pump the transition to phase-coherent states should be accompanied by spontaneous appearance of a linear polarization in the emission from the ground state. Consequently, linear polarization can be considered as an experimentally measurable order parameter of the polariton BEC.[37]

It is well known that the BKT transition between normal and superfluid states in two-dimensional systems is closely connected with the formation of the topological defects[38] (vortex-antivortex pairs). It is thus of a crucial importance to understand the structure and polarization properties of vortices in the homogeneous polariton condensates.

III. POLARIZATION VORTICES IN SPINOR POLARITON CONDENSATES

To the best of our knowledge, up to the present time, there exists only one theoretical work regarding vortices in the context of the spinor polariton condensation.[26] In this pioneer paper the polarization structure of the polariton vortices was analyzed and the existence of peculiar half-vortices was predicted[39] and those latter have been observed very recently.[40] It was shown that contrary to the case of a normal vortex in scalar superfluid, the particle density differs from zero in the center of a half-vortex. Besides, these objects have been predicted to possess a peculiar spatial dependence of the polarization: it is circular in the center of the core and becomes linear at large distances from it. The energy required to create a half-vortex is twice smaller than the one required to create a normal vortex because only one half of the total fluid mass is rotating. As a result, the existence of half-vortices as stationary stable states divides by 2 the critical temperature of the BKT phase transition as discussed in Ref. 26.

However, the effects of the in-plane effective magnetic fields of various nature, in particular, of the TE-TM splitting, on the structure of the polarization vortices was neglected in this seminal work. As we shall see below, these fields can have drastic effects on the structure of polarization vortices. Besides, in our opinion, the choice of the basis of linear polarizations used in Ref. 26 hindered the clear physical understanding of the nature of the half-vortices. In the present paper we revise and extend the results of Rubo, accounting for nonzero TE-TM splitting of a polariton doublet using the basis of circular polarizations, which makes the obtained results much more transparent.

The Hamiltonian of an interacting polariton system written in the basis of circular polarized states reads

$$\begin{aligned}\hat{H} &= \hat{H}_0 + \hat{H}_{int}\\ &= \int [\vec{\psi}^\dagger \hat{\mathbf{T}}(-i\hbar\nabla)\vec{\psi} - \mu(\vec{\psi}^\dagger\vec{\psi})]d\mathbf{r}\\ &\quad + \int \left[\frac{\alpha_1}{2}(|\psi_+|^4 + |\psi_-|^4) + \alpha_2|\psi_+|^2|\psi_-|^2\right]d\mathbf{r}, \end{aligned} \tag{3}$$

where $\psi_\pm$ are the field operators for right and left circular polarized polaritons, $\vec{\psi}=(\psi_+,\psi_-)^T$, the coefficients α_1 and α_2 describe the interaction between the polaritons with same and opposite circular polarizations,[41] and μ is the chemical potential determined by the condensate density at infinity. The parameters we use are connected with those introduced in Ref. 26 in the following way:

$$U_0 = \alpha_1, \tag{4}$$

$$U_1 = (\alpha_1 - \alpha_2)/2. \tag{5}$$

The tensor of the kinetic energy reads

$$\hat{\mathbf{T}}(-i\hbar\nabla) = \begin{pmatrix} \hat{H}_0(-i\hbar\nabla) & \hat{H}_{\text{TE-TM}}(-i\hbar\nabla) \\ \hat{H}^\dagger_{\text{TE-TM}}(-i\hbar\nabla) & \hat{H}_0(-i\hbar\nabla)\end{pmatrix}, \tag{6}$$

where the diagonal terms $\hat{H}_0$ describe the kinetic energy of lower cavity polaritons and the off-diagonal terms $\hat{H}_{\text{TE-TM}}$ correspond to the longitudinal-transverse splitting, mixing opposite circular polarized components. In our further considerations we will adopt the effective-mass approximation,

$$\hat{H}_0 = -\frac{\hbar^2}{2m^*}\nabla^2, \tag{7}$$

$$\hat{H}_{\text{TE-TM}} = \beta\left(\frac{\partial}{\partial y} + i\frac{\partial}{\partial x}\right)^2, \tag{8}$$

where m^* is the effective mass of cavity polaritons. Equation (8) is the simplest form of the Hamiltonian providing the correct symmetry of the effective magnetic field given by expression (2).[31,42] The dependence of the absolute value of this field on the wave number is taken to be quadratic, which corresponds well to the effective-mass approximation we are using in the current paper. β is a constant, characterizing the strength of the TE-TM splitting which can be expressed via the longitudinal and transverse polariton effective masses m_l and m_t,

$$\beta = \frac{\hbar^2}{4}\left(\frac{1}{m_l} - \frac{1}{m_t}\right). \tag{9}$$

Within the framework of mean-field approximation at $T=0$, the dynamics of the spinor polariton superfluid can be completely described by a set of 2 coupled Gross-Pitaevskii equations,[43] which in the basis of circular polarized states reads

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} = \begin{pmatrix} -\frac{\hbar^2}{2m^*}\nabla^2 - \mu + \alpha_1|\psi_+|^2 + \alpha_2|\psi_-|^2 & \beta\left(\frac{\partial}{\partial y} + i\frac{\partial}{\partial x}\right)^2 \\ \beta\left(\frac{\partial}{\partial y} - i\frac{\partial}{\partial x}\right)^2 & -\frac{\hbar^2}{2m^*}\nabla^2 - \mu + \alpha_1|\psi_-|^2 + \alpha_2|\psi_+|^2 \end{pmatrix}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}, \tag{10}$$

where the chemical potential is $\mu=(\alpha_1+\alpha_2)n_\infty/2$ with $n_\infty=|\psi_+(\infty)|^2+|\psi_-(\infty)|^2$ being the condensate density far away from the vortex core. Rescaling the variables $\psi_\pm \to [\mu/(\alpha_1+\alpha_2)]^{1/2}\psi_\pm$, $\mathbf{r}\to[\hbar^2/(m^*\mu)]^{1/2}\mathbf{r}$, and $t\to(\hbar/\mu)t$, one can represent the system (10) in the following dimensionless form:

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}=\begin{pmatrix}-\frac{1}{2}\nabla^2-1+A_1|\psi_+|^2+A_2|\psi_-|^2 & \chi\left(\frac{\partial}{\partial y}+i\frac{\partial}{\partial x}\right)^2\\ \chi\left(\frac{\partial}{\partial y}-i\frac{\partial}{\partial x}\right)^2 & -\frac{1}{2}\nabla^2-1+A_1|\psi_-|^2+A_2|\psi_+|^2\end{pmatrix}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}, \tag{11}$$

where $A_{1,2}=\alpha_{1,2}/(\alpha_1+\alpha_2)$ and $\chi=\beta m^*/\hbar^2$.

Let us start our analysis from the simplest case, where the TE-TM splitting can be neglected, $\chi=0$. This case has been considered already in Ref. 26 but we feel that it will be instructive to re-examine it using the basis of the circular polarized states because the final result is more transparent.

Equations (11) allow a time-independent solution, which can be represented in the following form:

$$\vec{\psi}=\begin{pmatrix}\psi_+(r,\theta)\\ \psi_-(r,\theta)\end{pmatrix}=\begin{pmatrix}f_+(r)e^{il_+\theta}\\ f_-(r)e^{il_-\theta}\end{pmatrix}, \tag{12}$$

where (r,θ) are the polar coordinates. Due to the conservation of the z component of the spin by polariton-polariton interactions, the winding numbers of the two circular polarized components $l_\pm$ are independent. Rewriting Eq. (12) in the basis of linear polarized components $\psi_\pm=2^{-1/2}(\psi_X\pm i\psi_Y)$, one easily obtains the relation between our winding numbers $l_\pm$ and those of Ref. 26,

$$k=\frac{l_+-l_-}{2}, \tag{13}$$

$$m=\frac{l_++l_-}{2}. \tag{14}$$

The situation describing a half-vortex corresponds to the case where for one circular polarized component the winding number is zero (say $l_+=0$) while for the other one it is ($l_-=+1$). Radial wave functions $f_\pm$ satisfy the following set of equations:

$$f_+''+f_+'+\left(2-2A_1f_+^2-2A_2f_-^2-\frac{l_+^2}{r^2}\right)f_+=0, \tag{15}$$

$$f_-''+f_-'+\left(2-2A_1f_-^2-2A_2f_+^2-\frac{l_-^2}{r^2}\right)f_-=0, \tag{16}$$

which corresponds to Eq. (10) of Ref. 26, if one puts $f_\pm=2^{-1/2}(f\pm g)$.

In the simplest case, when the circular polarized components do not interact ($A_2=0$), the half-vortex with $l_+=0$ and $l_-=1$ corresponds to a homogeneous distribution of σ_+ component and a simple vortex in σ_-. Clear enough, in the center of such a half-vortex the density is nonzero (due to the σ_+ component) and polarization is circular since the density of the σ_- component is zero in the center of the vortex. Moving from the center of the vortex changes polarization from circular to linear in a continuous manner.

Now let us consider a more interesting case where $\chi\neq 0$. The terms associated with the TE-TM splitting rewritten in polar coordinates read

$$\left(\frac{\partial}{\partial y}\pm i\frac{\partial}{\partial x}\right)^2=e^{\mp 2i\theta}\left(-\frac{\partial^2}{\partial r^2}\pm 2ir^{-1}\frac{\partial^2}{\partial r\,\partial\theta}\mp 2ir^{-2}\frac{\partial}{\partial\theta}+r^{-1}\frac{\partial}{\partial r}+r^{-2}\frac{\partial^2}{\partial\theta^2}\right). \tag{17}$$

The nonzero coupling between the circular polarized components leads to the mutual dependence of their winding numbers. The only cylindrically symmetric solutions of Eq. (11) have the following form:

$$\begin{pmatrix}\psi_+(r,\theta)\\ \psi_-(r,\theta)\end{pmatrix}=e^{il\theta}\begin{pmatrix}f_+(r)\\ e^{2i\theta}f_-(r)\end{pmatrix}, \tag{18}$$

which means that necessarily

$$l_+=l=l_--2. \tag{19}$$

In terms of Ref. 26 this state corresponds to a winding number $k=-1$. Thus, one can conclude that the presence of the TE-TM splitting does not allow the half-vortex as a stationary solution anymore.

The radial functions describing the vortex core can be found from the following system of coupled equations, which can be obtained by putting expressions (17) and (18) into Eq. (11),

$$\frac{1}{2}\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)f_+-\left(A_1f_+^2+A_2f_-^2-1+\frac{l^2}{2r^2}\right)f_++\chi\left[\frac{d^2}{dr^2}+\frac{2l+3}{r}\frac{d}{dr}+\frac{l(l+2)}{r^2}\right]f_-=0, \tag{20}$$

$$\frac{1}{2}\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)f_--\left[A_1f_-^2+A_2f_+^2-1+\frac{(l+2)^2}{2r^2}\right]f_-+\chi\left[\frac{d^2}{dr^2}-\frac{2l+1}{r}\frac{d}{dr}+\frac{l(l+2)}{r^2}\right]f_-=0. \tag{21}$$

The above equations are quite complicated and only allow numerical solution.

IV. RESULTS AND DISCUSSION

In this section we present numerical results for radial functions $f_\pm$ and the associated vortex polarization textures. To determine which configuration will have the lowest energy, let us remind that without the TE-TM splitting, the

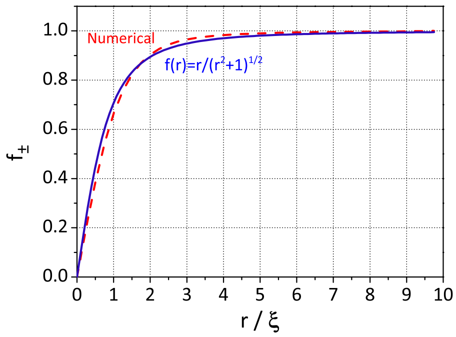

FIG. 1. (Color online) The exact numerical solution for radial function of $(-1,1)$ vortex (dashed red line) together with the fitting function (solid blue line). The parameter values are $\chi=-1/78$, $A_1=10/9$, and $A_2=-1/9$

elastic energy of the vortex in a spinor condensate can be estimated as[26]

$$E_{el}=\frac{\rho_s}{2}\int[(\nabla\theta_+)^2+(\nabla\theta_-)^2]d\mathbf{r}\approx\pi\rho_s(l_+^2+l_-^2)\ln\left(\frac{R}{a}\right) \tag{22}$$

where $\rho_s=\hbar^2 n_\infty/m^*$ is the rigidity or stiffness of the condensate, $a=\hbar/(m^*\mu)^{1/2}$ is the coherence length or the vortex core radius, R is the size of the system, and $\theta_\pm$ are the phases of the circular polarized components. From the above formula it follows that if $l_+=l_--2$, the minimal energy corresponds to a vortex $(l_+,l_-)=(-1,+1)$. We thus start our analysis from such a situation.

In this case the radial functions corresponding to the opposite circular polarizations found from numerical solution of Eqs. (20) and (21) with $l=-1$ are identical and can be satisfactory approximated by the following function plotted at Fig. 1:

$$f_\pm(r)\approx\frac{r}{\sqrt{r^2+1}}. \tag{23}$$

One should make a remark at this point. Indeed, if $f_+=f_-$ is a solution of Eqs. (20) and (21), this is also the case for $f_+=-f_-$ but in such a situation the pseudospin will point in the opposite direction with respect to the first case. We will talk again about this in the next section.

As $f_+=f_-$, the polarization of the system is always linear, which makes the z component of the pseudospin vanish: $S_z=0$. The pseudospin lies in the plane and is at any point aligned with the TE-TM effective field. The orientation of the TE-TM effective field depends on the wave-vector orientation, which, in turn, depends on the position of the particles with respect to the core of the vortex. The angular dependence both in reciprocal and real space is given by the formula (2), which shows that the orientation of the effective field varies as two times the polar angle φ. The resulting polarization pattern is one of a simple (with only one winding number) vortex with winding number 2, as it is shown in Fig. 2. If one investigates the $f_+=-f_-$ solution, the pseudospin will be totally symmetric and opposed to the TE-TM field, which is not only costly in terms of energy but also unstable to any perturbation.

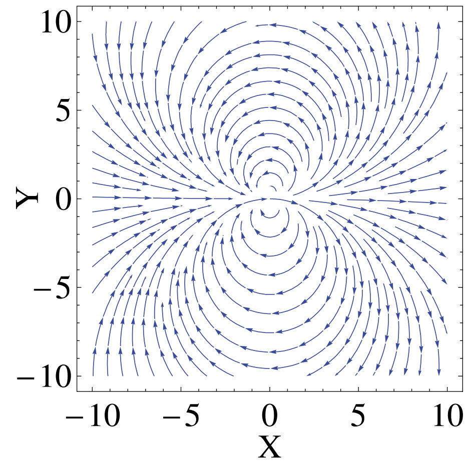

FIG. 2. (Color online) Pseudospin (S_x,S_y) vector field for $(-1,+1)$ configuration. This pattern is known to be the one of a simple vortex with winding number 2. The pseudospin is aligned with the TE-TM effective magnetic field.

Let us now try to understand qualitatively the way the TE-TM splitting would affect a half-vortex state, which could be created, for instance, by some external means. At $t=0$, σ_- particles are almost homogeneously covering space and immobile. They are not affected by the TE-TM splitting which is zero at $\vec{k}=0$. σ_+ particles are rotating. The pseudospin in the nonzero wave-vector states is mostly aligned along z, perpendicular to the TE-TM field which is in the plane. The pseudospin therefore starts to rotate, demonstrating that the half-vortex is not stationary. The speed of rotation is large close to the core where particles rotate fast and where the TE-TM splitting is large, whereas the rotation is slower and slower going away from the center. For each radius, the situation is reminiscent of the one happening in the optical spin Hall effect.[32] The density of σ_+ and σ_- particles is locally modified, which should provoke a drift of particles perpendicularly to the vortex motion and probably, a destruction of the vortex. The lifetime τ of such a transient state is therefore linked with the value of the TE-TM splitting in the core region. We propose an estimation based on the strength of the energy splitting β seen by particles moving at the core radius $\xi=\hbar/\sqrt{m^*\mu}$ characterized by a wave vector $k=1/\xi$,

$$\tau=\frac{\hbar\xi^2}{\beta}. \tag{24}$$

This value can strongly depend on the type of structure, on the value of detuning, etc. The typical values which can be expected, however, lie between ten and a few hundreds of picoseconds. These times are comparable to the typical coherence times which have been measured for polariton condensates. We conclude that the half-vortices could be experimentally observed both in resonant and nonresonant experiments. They are however, intrinsically transient states with a lifetime probably limited by the TE-TM splitting

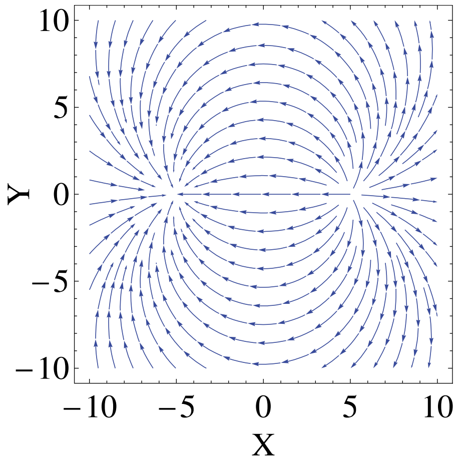

FIG. 3. (Color online) Pseudospin (S_x,S_y) vector field for a separation $d=5$ of the σ_+ and σ_- vortices along the x axis.

value. Thus they should not be considered, in principle, in a rigorous calculation of the BKT critical temperature.

The vortex $(-1,+1)$ can be considered as a bound state of two half-vortices, $(-1,0)$ and $(0,+1)$. As it was shown in Ref. 26, without TE-TM splitting the interaction energy of a pair of vortices (l_{1+},l_{1-}) and (l_{2+},l_{2-}) placed at distance d from each other reads

$$E_{int} \approx 2\pi\rho_s(k_1k_2+m_1m_2)\ln(a/d) = \pi\rho_s(l_{1+}l_{2+} + l_{1-}l_{2-})\ln(a/d). \tag{25}$$

According to this formula, the half-vortices $(-1,0)$ and $(0,+1)$ do not interact and the corresponding vortex pair is unbound. Let us re-examine the $(-1,+1)$ case while adding the distance d between σ_+ and σ_- vortices along the x axis. One has to write the associated wave function in Cartesian coordinates with (23), $r=\sqrt{x^2+y^2}$, and $\theta_\pm=\arctan(y/x)+\pi H(-x)$ (H is the Heaviside function),

$$\begin{pmatrix}\psi_+(x+d,y)\\ \psi_-(x-d,y)\end{pmatrix} = \begin{pmatrix} f_+(x+d,y)e^{il_+\theta_+(x+d,y)} \\ f_-(x-d,y)e^{il_-\theta_-(x-d,y)}\end{pmatrix}. \tag{26}$$

The corresponding pseudospin configuration is shown in Fig. 3 for $d=5$ and the pattern compared with one of Fig. 2 shows that as d increases, the pseudospin becomes less and less aligned with the TE-TM field and the energy should consequently increase. The normalized TE-TM energy part of the polariton condensate reads

$$E_{\text{TE-TM}} = \rho_s\chi\int\left[\psi_+^*\left(\frac{\partial}{\partial y}+i\frac{\partial}{\partial x}\right)^2\psi_- + \psi_-^*\left(\frac{\partial}{\partial y}-i\frac{\partial}{\partial x}\right)^2\psi_+\right]d\mathbf{r}. \tag{27}$$

The numerical computation of $E_{\text{TE-TM}}$ as a function of d is shown in Fig. 4. One can see that the energy increases logarithmically with d and that the lowest energy state is, as expected, the one with $d=0$, where the pseudospin is aligned with the effective field. Thus the TE-TM splitting makes $(-1,0)$ and $(0,+1)$ vortices interact and collapse on each other to form a $(-1,+1)$ vortex. Let us remark that for the $f_+=-f_-$ solution, the opposite behavior will be observed and

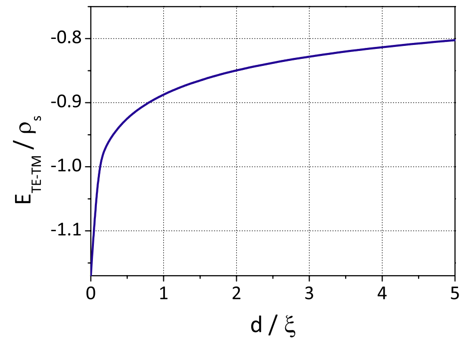

FIG. 4. (Color online) TE-TM normalized energy part as a function of the separation d in units of ξ, the lowest energy is reached for $d=0$.

the energy will decrease when vortices move away from each other. For $d=0$ and a system of size πR^2, the TE-TM and the kinetic energy read

$$E_{kin} = \rho_s\frac{\pi}{2}\left[\frac{R^2(R^2-2)}{(1+R^2)^2}+2\ln(1+R^2)\right], \tag{28}$$

$$E_{\text{TE-TM}} = 2\chi E_{kin}, \tag{29}$$

which with Eq. (9), $m^{*-1}=2^{-1}(m_t^{-1}+m_l^{-1})$ and the definitions of χ and ρ_s give $E_c+E_{\text{TE-TM}}=E_c^*$, where E_c^* is the kinetic energy associated with the new rigidity constant $\rho_s^*=n_\infty\hbar^2/m_t$. One concludes, as it could be expected, that the TE-TM effective magnetic field switches the effective mass m^* to the TE polarized particles mass m_t.[44]

Finally we will say a word about $(0,+2)$ and $(+1,+3)$ configurations that, if they are not energetically favorable, exhibit interesting pseudospin (polarization) patterns. One can note, by the way, that these two states are totally symmetric respectively to $(-2,0)$ and $(-3,-1)$. In these cases radial functions are no more identical for the two compo-

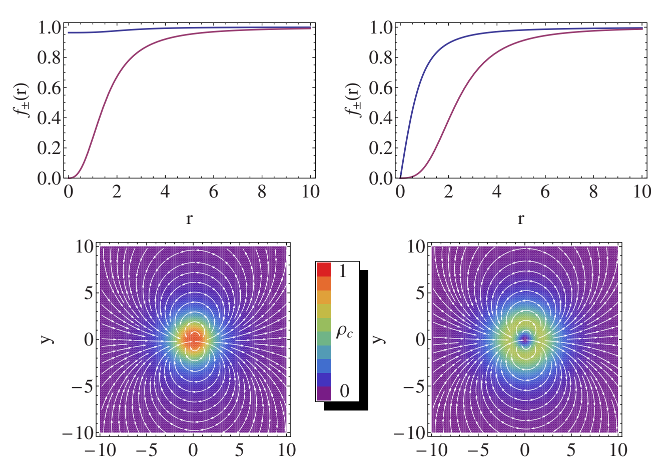

FIG. 5. (Color online) Left column is $(0,+2)$ and right column is $(+1,+3)$ configuration. The top line shows radial functions with a blue line for f_+ and a purple line for f_-. The bottom line shows $[S_x(x,y),S_y(x,y)]$ vector fields (white arrows) over $\rho_c=2S_z(x,y)$ (the degree of circular polarization) background.

nents, which will introduce a nonzero circular polarization close to the vortex core. Numerically calculated radial functions are plotted in the upper part of Fig. 5 and $S_z(x,y)$ functions as the background of (S_x,S_y) vector fields in the lower part. One can remark that the latter vector fields are exactly the same as the one in Fig. 2. Indeed, this configuration is fixed by the condition in Eq. (19). The $(0,+2)$ state is peculiar, so far as there is no vortex for σ_+. Nevertheless, the corresponding radial function is not constant as expected. Indeed, the interaction between circular components implies a small depletion around the center of the system, observed as a minimum at $r=0$. The polarization becomes more and more circular while approaching $r=0$. In the $(+1,+3)$ configuration, one has a vortex for each component and the pseudospin S_z component exhibits a maximum before reaching $r=0$ which corresponds to a ring around the vortex core with the maximum of circular polarization degree at about $r=0.6$.

V. CONCLUSIONS

In conclusion, we analyzed the impact of the TE-TM splitting on vortices in spinor polariton condensates. We have shown that this splitting induces a qualitative change in the nature of the stationary vortex state supported by a polariton condensate. The half-vortices are no more stationary solutions of the spinor Gross-Pitaevskii equations and should not affect the critical temperature of the BKT phase transition. Their lifetime is of the order of ten to a few hundreds of picoseconds, limited by the TE-TM splitting value. However, they can, in principle, be observed experimentally. The stable vortex having the smallest energy is the state $(-1,+1)$ (in the circular basis), whose polarization pattern follows the one implied by the peculiar TE-TM symmetry. Polarization textures of other vortex states $[(0,+2)$ and $(+1,+3)]$ have also been analyzed.

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